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Introduction to Real Analysis is a comprehensive book for undergraduate student s of Mathematics. The book comprises chapters on preliminaries, the real numbers, sequences and states, limits, continuous functions, differentiation, infinite series, and the generalized Reimann Integral. In addition, the book consists of several chapterwise problems to understand the concepts better. This book is essential for students who aspire to learn the basic concepts and techniques of real analysis. About Wiley India Pvt. Ltd. Wiley India Pvt. Ltd. is the Indian chapter of John Wiley & Sons, Inc., which is a leading publisher globally. They develop and publish quality and scholarly books, journals, online content, reference works, and are also into certification and training services, and online applications among many other things. Wiley India Pvt. Ltd. is publishing quality educational and professional books. Some of the books published under their banner are Principles of Physics, Operating System Principles, Calculus, An Introduction to Probability and Statistics, and Gravitation and Cosmology. By: Jiří Lebl (website #1 (personal), website #2 (work: OSU), email:) Jump to: [Download the book (volume I) as PDF] [volume II as PDF] [Buy paperback (volume I) on Amazon] [volume II on Amazon] [Web version] [Search] This free online textbook (OER more formally) is a course in undergraduate real analysis (somewhere it is called "advanced calculus"). The book is meant both for a basic course for students who do not necessarily wish to go to graduate school, but also as a more advanced course that also covers topics such as metric spaces and should prepare students for graduate study. A prerequisite for the course is a basic proof course. An advanced course could be two semesters long with some of the second-semester topics such as multivariable differential calculus, path integrals, and the multivariable integral using the second volume. There are more topics than can be covered in two semesters, and it can also be reading for beginning graduate students to refresh their analysis or fill in some of the holes. This book started its life as my lecture notes for Math 444 at the University of Illinois at Urbana-Champaign (UIUC) in the fall semester of 2009. It was later enhanced to teach the Math 521/522 sequence at University of Wisconsin-Madison (UW-Madison) and the Math 4143/4153 sequence at Oklahoma State University (OSU). The book (volume I) starts with analysis on the real line, going through sequences, series, and then into continuity, the derivative, and the Riemann integral using the Darboux approach. There are plenty of available detours along the way, or we can power through towards the metric spaces in chapter 7. The philosophy is that metric spaces are absorbed much better by the students after they have gotten comfortable with basic analysis techniques in the very concrete setting of the real line. As a bonus, the book can be used both by a slower-paced, less abstract course, and a faster-paced more abstract course for future graduate students. The slower course never reaches metric spaces. A nice capstone theorem for such a course is the Picard theorem on existence and uniqueness of ordinary differential equations, a proof which brings together everything one has learned in the course. A faster-paced course would generally reach metric spaces, and as a reward such students can see a streamlined (but more abstract) proof of Picard. Volume II continues into multivariable analysis. Starting with differential calculus, including inverse and implicit function theorems, continuing with differentiation under the integral and path integrals, which are often not covered in a course like this, and multivariable Riemann integral. Finally, there is also a chapter on power series, Arzelà-Ascoli, Stone-Weierstrass, and Fourier series. Together the two volumes provide enough material for several different types of year-long sequences. A student who absorbs the first volume and the first three chapters of volume II should be more than prepared for graduate real and complex analysis courses. I have tried (especially in recent editions) to add many diagrams and graphs to graphically illustrate the proofs and make them more accessible. Usually, these are precise and more in-depth versions of the drawings I attempt on the board in class. Together, the two volumes have over a hundred figures. The aim is to provide a low cost, redistributable, not overly long, high-quality textbook that students will actually keep rather than selling back after the semester is over. Even if the students throw it out, they can always look it up on the net again. You are free to have a local bookstore or copy store make and sell copies for your students. See below about the license. One reason for making the book freely available is to allow modification and customization for a specific purpose if necessary (as the University of Pittsburgh has done for example). If you do modify this book, make sure to mark them prominently as such to avoid confusion. This aspect is also important for the longevity of the book. The book can be updated and modified even if I happen to drop off the face of the earth. You do not have to depend on any publisher being interested as with traditional textbooks. Furthermore, errata are fixed promptly, meaning that if you teach the same class next term, all errata that are spotted are most likely already fixed. No need to wait several years for a new edition. Every once in a while I make some major addition and a new major version (edition), and then in between as errata are fixed I make minor version updates (like a corrected printing) usually once or twice a year, depending on the errata discovered. Exercise, chapter, and section numbers are preserved as much as humanly possible. What's added is added at the end with new numbers, so the book is generally compatible even if students (or the instructor) have an older printed copy. The minor updates are totally interchangeable and have very minimal changes, essentially nothing new. MAA published a review of the book (they looked at the December 2012 edition of Volume I, there was only the first volume then). Table of contents: Introduction 1. Real Numbers 2. Sequences and Series 3. Continuous Functions 4. The Derivative 5. The Riemann Integral 6. Sequences of Functions 7. Metric Spaces Volume II: 8. Several Variables and Partial Derivatives 9. One Dimensional Integrals in Several Variables 10. Multivariable Integral 11. Functions as Limits There are 528 exercises and 65 figures in Volume I (version 5.4, that is, June 8th 2021 edition). There are 263 exercises and 43 figures in Volume II (version 2.4, that is, June 8th 2021 edition). Please let me know at if you find any typos or have corrections, extra exercises or material, or any other comments. There is no solutions manual for the exercises. This situation is intentional. There is an unfortunately large number of problems with solutions out there already. Part of learning how to do proofs is to learn how to recognize your proof is correct. Looking at someone else's proof is a far less effective way of checking your proof than actually checking your proof. It is like going the gym and watching other people work out. The exercises in the book are meant to be a gym for the mind. If you are unsure about the correctness of a solution, then you do not yet have a solution. Furthermore, the best solution for the student is the one that the student comes up with on their own, not necessarily the one that the professor or the book author comes up with. Adoption: Do let me know () if you use the book for teaching a course! The book was used, or is being used, as the primary textbook at (other than my courses at UIUC, UCSD, UW-Madison, and OSU) University of California at Berkeley, University of Pittsburgh, Vancouver Island University, Western Illinois University, Medgar Evers College, San Diego State University, University of Toledo, Oregon Institute of Technology, Iowa State University, California State University Dominguez Hills, St. John's University of Tanzania, Mary Baldwin College, Ateneo De Manila University, University of New Brunswick Saint John, and many others. See below for a more complete list. The book has been selected as an Approved Textbook in the American Institute of Mathematics Open Textbook Initiative. See a list of classroom adoptions for more details. Download: Download the volume I of the book as PDF (Version 5.4, June 8th, 2021, 282 pages, 1.8 MB download) Download the volume II of the book as PDF (Version 2.4, June 8th, 2021, 195 pages, 1.4 MB download) Check for any errata (volume I) (volume II) in the current version. Look at the change log (volume I) (volume II) to see what changed in the newest version. I started numbering things with version numbers starting at 4.0 for volume I, and version 1.0 for volume II. The first number is the major number and it really means "edition" and will be raised when substantial changes are made. The second number is raised for corrections only. Buy paperback: I get a bit of money when you buy these (depending on where exactly they are bought). Probably enough to buy me a coffee (as long as it is not a fancy coffee), so by buying a copy you will support this project. You will also save your toner cartridge. 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You can get an archive of the source of the released version on github, look under though if you plan to work with it, maybe best to look at just the latest working version as that might have errata fixes or new additions. On the other hand, this might be a work in progress. Just ask me if unsure. Volume I is realanal.tex and volume II is realanal2.tex (those are the "driver files" text is in separate files for each chapter). I compile the pdf with pdflatex. You need to compile the first volume first before the second volume. You might need to run makeindex (for the index) and makeglossary (for the list of notations) as well, though theoretically it should now be handled automatically. There are scripts publish.sh and publish2.sh, that run everything an obnoxious number of times to make sure it all works. During the writing of this book, the author was in part supported by NSF grant DMS-0900885 and DMS-1362337. 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Trench, Introduction to real analysis, Pearson Education, 2003, . You're Reading a Free Preview Page 4 is not shown in this preview. 4th Edition Donald R. Sherbert, Robert ... 4th Edition Donald R. Sherbert, Robert ... 4th Edition Donald R. Sherbert, Robert ... 4th Edition Donald R. Sherbert, Robert ... 2nd Edition Robert G. Bartle 1. CONTENTS Introduction to Real Analysis 4th Edition Bartle Solutions Manual Full Download: This sample only, Download all chapters at: alibabadownload.com 2. CHAPTER 1 PRELIMINARIES We suggest that this chapter be treated as review and covered quickly, without detailed classroom discussion. For one reason, many of these ideas will be already familiar to the students — at least informally. Further, we believe that, in practice, those notions of importance are best learned in the arena of real analysis, where their use and significance are more apparent. Dwelling on the formal aspect of sets and functions does not contribute very greatly to the students' understanding of real analysis. If the students have already studied abstract algebra, number theory or com- binatorics, they should be familiar with the use of mathematical induction. If not, then some time should be spent on mathematical induction. The third section deals with finite, infinite and countable sets. These notions are important and should be briefly introduced. However, we believe that it is not necessary to go into the proofs of these results at this time. Section 1.1 Students are usually familiar with the notations and operations of set algebra, so that a brief review is quite adequate. One item that should be mentioned is that two sets A and B are often proved to be equal by showing that: (i) if x ∈ A, then x ∈ B, and (ii) if x ∈ B, then x ∈ A. This type of element-wise argument is very common in real analysis, since manipulations with set identities is often not suitable when the sets are complicated. Students are often not familiar with the notions of functions that are injective (= one-one) or surjective (= onto). Sample Assignment: Exercises 1, 3, 9, 14, 15, 20. Partial Solutions: 1. (a) B ∩ C = {5, 11, 17, 23, . . .} = {6k − 1 : k ∈ N}, A ∩ (B ∩ C) = {5, 11, 17} (b) (A ∩ B) C = {2, 8, 14, 20} (c) (A ∩ C) B = {3, 7, 9, 13, 15, 19} 2. The sets are equal to (a) A, (b) A ∩ B, (c) the empty set. 3. If A ⊆ B, then x ∈ A implies x ∈ B, whence x ∈ A∩B, so that A ⊆ A ∩ B ⊆ A. Thus, if A ⊆ B, then A = A ∩ B. Conversely, if A = A ∩ B, then x ∈ A implies x ∈ A ∩ B, whence x ∈ B. Thus if A = A ∩ B, then A ⊆ B. 4. If x is in A (B ∩ C), then x is in A but x /∈ B ∩ C, so that x ∈ A and x is either not in B or not in C. Therefore either x ∈ A B or x ∈ A C, which implies that x ∈ (A B) ∪ (A C). Thus A (B ∩ C) ⊆ (A B) ∪ (A C). 1 3. 2 Bartle and Sherbert Conversely, if x is in (A B) ∪ (A C), then x ∈ A B or x ∈ A C. Thus x ∈ A and either x /∈ B or x /∈ C, which implies that x ∈ A but x /∈ B ∩ C, so that x ∈ A (B ∩ C). Thus (A B) ∪ (A C) ⊆ A (B ∩ C). Since the sets A (B ∩C) and (A B)∪(A C) contain the same elements, they are equal. 5. (a) If x ∈ A ∩ (B ∪ C), then x ∈ A and x ∈ B ∪ C. Hence we either have (i) x ∈ A and x ∈ B, or we have (ii) x ∈ A and x ∈ C. Therefore, either x ∈ A ∩ B or x ∈ A ∩ C, so that x ∈ (A ∩ B) ∪ (A ∩ C). This shows that A ∩ (B ∪ C) is a subset of (A ∩ B) ∪ (A ∩ C). Conversely, let y be an element of (A ∩ B) ∪ (A ∩ C). Then either (j) y ∈ A ∩ B, or (jj) y ∈ A ∩ C. It follows that y ∈ A and either y ∈ B or y ∈ C. Therefore, y ∈ A and y ∈ B ∪ C, so that y ∈ A ∩ (B ∪ C). Hence (A ∩ B) ∪ (A ∩ C) is a subset of A ∩ (B ∪ C). In view of Definition 1.1.1, we conclude that the sets A ∩ (B ∪ C) and (A ∩ B) ∪ (A ∩ C) are equal. (b) Similar to (a). 6. The set D is the union of {x : x ∈ A and x /∈ B} and {x : x /∈ A and x ∈ B}. 7. Here An = {n + 1, 2(n + 1), . . .}. (a) A1 = {2, 4, 6, 8, . . .}, A2 = {3, 6, 9, 12, . . .}, A1 ∩ A2 = {6, 12, 18, 24, . . .} = {6k : k ∈ N} = A5. (b) An = N {1}, because if n > 1, then n ∈ An−1; moreover 1 /∈ An. Also An = ∅, because n /∈ An for any n ∈ N. 8. (a) The graph consists of four horizontal line segments. (b) The graph consists of three vertical line segments. 9. No. For example, both (0, 1) and (0, − 1) belong to C. 10. (a) f(E) = {1/x2 : 1 ≤ x ≤ 2} = {y : 1 4 ≤ y ≤ 1} = [1 4 , 1]. (b) f−1(G) = {x : 1 ≤ 1/x2 ≤ 4} = {x : 1 4 ≤ x2 ≤ 1} = [−1, −1 2] ∪ [1 2 , 1]. 11. (a) f(E) = {x + 2 : 0 ≤ x ≤ 1} = [2, 3], so h(E) = g(f(E)) = g([2, 3]) = {y2 : 2 ≤ y ≤ 3} = [4, 9]. (b) g−1(G) = {y : 0 ≤ y2 ≤ 4} = [−2, 2], so h−1(G) = f−1(g−1(G)) = f−1([−2, 2]) = {x : −2 ≤ x + 2 ≤ 2} = [−4, 0]. 12. If 0 is removed from E and F, then their intersection is empty, but the intersection of the images under f is {y : 0 < y ≤ 1}. 13. E F = {x : −1 ≤ x < 0}, f(E) f(F) is empty, and f(E F) = {y : 0 < y ≤ 1}. 14. If y ∈ f(E ∩ F), then there exists x ∈ E ∩ F such that y = f(x). Since x ∈ E implies y ∈ f(E), and x ∈ F implies y ∈ f(F), we have y ∈ f(E) ∩ f(F). This proves f(E ∩ F) ⊆ f(E) ∩ f(F). 15. If x ∈ f−1(G) ∩ f−1(H), then x ∈ f−1(G) and x ∈ f−1(H), so that f(x) ∈ G and f(x) ∈ H. Then f(x) ∈ G ∩ H, and hence x ∈ f−1(G ∩ H). This shows 4. Chapter 1 — Preliminaries 3 that f−1(G) ∩ f−1(H) ⊆ f−1(G ∩ H). The opposite inclusion is shown in Example 1.1.8(b). The proof for unions is similar. 16. If f(a) = f(b), then a/ √ a2 + 1 = b/ √ b2 + 1, from which it follows that a2 = b2. Since a and b must have the same sign, we get a = b, and hence f is injective. If −1 < y < 1, then x := y/ 1 − y2 satisfies f(x) = y (why?), so that f takes R onto the set {y : − 1 < y < 1}. If x > 0, then x = √ x2 < √ x2 + 1, so it follows that f(x) ∈ {y : 0 < y < 1}. 17. One bijection is the familiar linear function that maps a to 0 and b to 1, namely, f(x) := (x − a)/(b − a). Show that this function works. 18. (a) Let f(x) = 2x, g(x) = 3x. (b) Let f(x) = x2, g(x) = x, h(x) = 1. (Many examples are possible.) 19. (a) If x ∈ f−1(f(E)), then f(x) ∈ f(E), so that there exists x1 ∈ E such that f(x1) = f(x). If f is injective, then x1 = x, whence x ∈ E. Therefore, f−1(f(E)) ⊆ E. Since E ⊆ f−1(f(E)) holds for any f, we have set equality when f is injective. See Example 1.1.8(a) for an example. (b) If y ∈ H and f is surjective, then there exists x ∈ A such that f(x) = y. Then x ∈ f−1(H) so that y ∈ f(f−1(H)). Therefore H ⊆ f(f−1(H)). Since f(f−1(H)) ⊆ H for any f, we have set equality when f is surjective. See Example 1.1.8(a) for an example. 20. (a) Since y = f(x) if and only if x = f−1(y), it follows that f−1(f(x)) = x and f(f−1(y)) = y. (b) Since f is injective, then f−1 is injective on R(f). And since f is surjec- tive, then f−1 is defined on R(f) = B. 21. If g(f(x1)) = g(f(x2)), then f(x1) = f(x2), so that x1 = x2, which implies that g ◦ f is injective. If w ∈ C, there exists y ∈ B such that g(y) = w, and there exists x ∈ A such that f(x) = y. Then g(f(x)) = w, so that g ◦ f is surjective. Thus g ◦ f is a bijection. 22. (a) If f(x1) = f(x2), then g(f(x1)) = g(f(x2)), which implies x1 = x2, since g ◦ f is injective. Thus f is injective. (b) Given w ∈ C, since g ◦ f is surjective, there exists x ∈ A such that g(f(x)) = w. If y := f(x), then y ∈ B and g(y) = w. Thus g is surjective. 23. We have x ∈ f−1(g−1(H)) ⇐⇒ f(x) ∈ g−1(H) ⇐⇒ g(f(x)) ∈ H ⇐⇒ x ∈ (g ◦ f)−1(H). 24. If g(f(x)) = x for all x ∈ D(f), then g ◦ f is injective, and Exercise 22(a) implies that f is injective on D(f). If f(g(y)) = y for all y ∈ D(g), then Exercise 22(b) implies that f maps D(f) onto D(g). Thus f is a bijection of D(f) onto D(g), and g = f−1. Section 1.2 The method of proof known as Mathematical Induction is used frequently in real analysis, but in many situations the details follow a routine patterns and are 5. 4 Bartle and Sherbert left to the reader by means of a phrase such as: "The proof is by Mathematical Induction". Since may students have only a hazy idea of what is involved, it may be a good idea to spend some time explaining and illustrating what constitutes a proof by induction. Pains should be taken to emphasize that the induction hypothesis does not entail "assuming what is to be proved". The inductive step concerns the validity of going from the assertion for k ∈ N to that for k + 1. The truth of falsity of the individual assertion is not an issue here. Sample Assignment: Exercises 1, 2, 6, 11, 13, 14, 20. Partial Solutions: 1. The assertion is true for n = 1 because 1/(1 · 2) = 1/(1 + 1). If it is true for n = k, then it follows for k + 1 because k/(k + 1) + 1/[(k + 1)(k + 2)] = (k + 1)/(k + 2). 2. The statement is true for n = 1 because [1 2 · 1 · 2]2 = 1 = 13. For the inductive step, use the fact that 1 2 k(k + 1) 2 + (k + 1)3 = 1 2 (k + 1)(k + 2) 2 . 3. It is true for n = 1 since 3 = 4 − 1. If the equality holds for n = k, then add 8(k + 1) − 5 = 8k + 3 to both sides and show that (4k2 − k) + (8k + 3) = 4(k + 1)2 − (k + 1) to deduce equality for the case n = k + 1. 4. It is true for n = 1 since 1 = (4 − 1)/3. If it is true for n = k, then add (2k + 1)2 to both sides and use some algebra to show that 1 3 (4k3 − k) + (2k + 1)2 = 1 3 [4k3 + 12k2 + 11k + 3] = 1 3 [4(k + 1)3 − (k + 1)], which establishes the case n = k + 1. 5. Equality holds for n = 1 since 12 = (−1)2(1 · 2)/2. The proof is completed by showing (−1)k+1[k(k + 1)]/2 + (−1)k+2(k + 1)2 = (−1)k+2[(k + 1)(k + 2)]/2. 6. If n = 1, then 13 + 5 · 1 = 6 is divisible by 6. If k3 + 5k is divisible by 6, then (k + 1)3 + 5(k + 1) = (k3 + 5k) + 3k(k + 1) + 6 is also, because k(k + 1) is always even (why?) so that 3k(k + 1) is divisible by 6, and hence the sum is divisible by 6. 7. If 52k − 1 is divisible by 8, then it follows that 52(k+1) − 1 = (52k − 1) + 24 · 52k is also divisible by 8. 8. 5k+1 −4(k +1)−1=5·5k −4k − 5 = (5k − 4k − 1) + 4(5k − 1). Now show that 5k − 1 is always divisible by 4. 9. If k3 + (k + 1)3 + (k + 2)3 is divisible by 9, then (k + 1)3 + (k+2)3 + (k + 3)3 = k3 + (k + 1)3 + (k + 2)3 + 9(k2 + 3k + 3) is also divisible by 9. 10. The sum is equal to n/(2n + 1). 6. Chapter 1 — Preliminaries 5 11. The sum is 1 + 3 + · · · + (2n − 1) = n2. Note that k2 + (2k + 1) = (k + 1)2. 12. If n0 > 1, let S1 := {n ∈ N : n − n0 + 1 ∈ S} Apply 1.2.2 to the set S1. 13. If k < 2k, then k + 1 < 2k + 1 < 2k + 2k = 2(2k) = 2k + 1. 14. If n = 4, then 24 = 16 < 24 = 4!. If 2k < k! and if k ≥ 4, then 2k+1 = 2 · 2k < 2 · k! < (k + 1) · k! = (k + 1)!. [Note that the inductive step is valid when- ever 2 < k + 1, including k = 2, 3, even though the statement is false for these values.] 15. For n = 5 we have 7 ≤ 23. If k ≥ 5 and 2k − 3 ≤ 2k−2, then 2(k + 1) − 3 = (2k − 3) + 2 ≤ 2k−2 + 2k−2 = 2(k + 1)−2. 16. It is true for n = 1 and n ≥ 5, but false for n = 2, 3, 4. The inequality 2k + 1 < 2k, wich holds for k ≥ 3, is needed in the induction argument. [The inductive step is valid for n = 3, 4 even though the inequality n2 < 2n is false for these values.] 17. m = 6 trivially divides n3 − n for n = 1, and it is the largest integer to divide 23 − 2 = 6. If k3 − k is divisible by 6, then since k2 + k is even (why?), it follows that (k + 1)3 − (k + 1) = (k3 − k) + 3(k2 + k) is also divisible by 6. 18. √ k + 1/ √ k + 1 = (√ k √ k + 1 + 1)/ √ k + 1 > (k + 1)/ √ k + 1 = √ k + 1. 19. First note that since 2 ∈ S, then the number 1 = 2 − 1 belongs to S. If m /∈ S, then m < 2m ∈ S, so 2m − 1 ∈ S, etc. 20. If 1 ≤ xk−1 ≤ 2 and 1 ≤ xk ≤ 2, then 2 ≤ xk−1 + xk ≤ 4, so that 1 ≤ xk + 1 = (xk−1 + xk)/2 ≤ 2. Section 1.3 Every student of advanced mathematics needs to know the meaning of the words "finite", "infinite", "countable" and "uncountable". For most students at this level it is quite enough to learn the definitions and read the statements of the theorems in this section, but to skip the proofs. Probably every instructor will want to show that Q is countable and R is uncountable (see Section 2.5). Some students will not be able to comprehend that proofs are necessary for "obvious" statements about finite sets. Others will find the material absolutely fascinating and want to prolong the discussion forever. The teacher must avoid getting bogged down in a protracted discussion of cardinal numbers. Sample Assignment: Exercises 1, 5, 7, 9, 11. Partial Solutions: 1. If T1 = ∅ is finite, then the definition of a finite set applies to T2 = Nn for some n. If f is a bijection of T1 onto T2, and if g is a bijection of T2 onto Nn, then (by Exercise 1.1.21) the composite g ◦ f is a bijection of T1 onto Nn, so that T1 is finite. 7. 6 Bartle and Sherbert 2. Part (b) Let f be a bijection of Nm onto A and let C = {f(k)} for some k ∈ Nm. Define g on Nm−1 by g(i) := f(i) for i = 1, . . . , k − 1, and g(i) := f(i + 1) for i = k, . . . , m − 1. Then g is a bijection of Nm−1 onto AC. (Why?) Part (c) First note that the union of two finite sets is a finite set. Now note that if C/B were finite, then C = B ∪ (C B) would also be finite. 3. (a) The element 1 can be mapped into any of the three elements of T, and 2 can then be mapped into any of the two remaining elements of T, after which the element 3 can be mapped into only one element of T. Hence there are 6 = 3 · 2 · 1 different injections of S into T. (b) Suppose a maps into 1. If b also maps into 1, then c must map into 2; if b maps into 2, then c can map into either 1 or 2. Thus there are 3 surjections that map a into 1, and there are 3 other surjections that map a into 2. 4. f(n) := 2n + 13, n ∈ N. 5. f(1) := 0, f(2n) := n, f(2n + 1) := −n for n ∈ N. 6. The bijection of Example 1.3.7(a) is one example. Another is the shift defined by f(n) := n + 1 that maps N onto N {1}. 7. If T1 is denumerable, take T2 = N. If f is a bijection of T1 onto T2, and if g is a bijection of T2 onto N, then (by Exercise 1.1.21) g ◦ f is a bijection of T1 onto N, so that T1 is denumerable. 8. Let An := {n} for n ∈ N, so An = N. 9. If S∩T = ∅ and f : N → S, g: N → T are bijections onto S and T, respectively, let h(n) := f((n + 1)/2) if n is odd and h(n) := g(n/2) if n is even. It is readily seen that h is a bijection of N onto S ∪ T; hence S ∪ T is denumerable. What if S ∩ T = ∅? 10. (a) m + n − 1 = 9 and m = 6 imply n = 4. Then h(6, 4) = 1 2 · 8 · 9 + 6 = 42. (b) h(m, 3) = 1 2 (m + 1)(m + 2) + m = 19, so that m2 + 5m − 36 = 0. Thus m = 4. 11. (a) P({1, 2}) = {∅, {1}, {2}, {1, 2}} has 22 = 4 elements. (b) P({1, 2, 3}) has 23 = 8 elements. (c) P({1, 2, 3, 4}) has 24 = 16 elements. 12. Let Sn+1 := {x1, . . . , xn, xn+1} = Sn ∪ {xn+1} have n + 1 elements. Then a subset of Sn+1 either (i) contains xn+1, or (ii) does not contain xn+1. The induction hypothesis implies that there are 2n subsets of type (i), since each such subset is the union of {xn+1} and a subset of Sn. There are also 2n subsets of type (ii). Thus there is a total of 2n + 2n = 2 · 2n = 2n + 1 subsets of Sn+1. 13. For each m ∈ N, the collection of all subsets of Nm is finite. (See Exercise 12.) Every finite subset of N is a subset of Nm for a sufficiently large m. Therefore Theorem 1.3.12 implies that F(N) = ∞ m=1 P(Nm) is countable. 8. CHAPTER 2 THE REAL NUMBERS Students will be familiar with much of the factual content of the first few sections, but the process of deducing these facts from a basic list of axioms will be new to most of them. The ability to construct proofs usually improves gradually during the course, and there are much more significant topics forthcoming. A few selected theorems should be proved in detail, since some experience in writing formal proofs is important to students at this stage. However, one should not spend too much time on this material. Sections 2.3 and 2.4 on the Completeness Property form the heart of this chapter. These sections should be covered thoroughly. Also the Nested Intervals Property in Section 2.5 should be treated carefully. Section 2.1 One goal of Section 2.1 is to acquaint students with the idea of deducing conse- quences from a list of basic axioms. Students who have not encountered this type of formal reasoning may be somewhat uncomfortable at first, since they often regard these results as "obvious". Since there is much more to come, a sampling of results will suffice at this stage, making it clear that it is only a sampling. The classic proof of the irrationality of √ 2 should certainly be included in the discussion, and students should be asked to modify this argument for √ 3, etc. Sample Assignment: Exercises 1(a,b), 2(a,b), 3(a,b), 6, 13, 16(a,b), 20, 23. Partial Solutions: 1. (a) Apply appropriate algebraic properties to get b = 0 + b = (−a + a) + b = −a + (a + b) = −a + 0 = −a. (b) Apply (a) to (−a) + a = 0 with b = a to conclude that a = −(−a). (c) Apply (a) to the equation a + (−1)a = a(1 + (−1)) = a · 0 = 0 to conclude that (−1)a = −a. (d) Apply (c) with a = −1 to get (−1)(−1) = −(−1). Then apply (b) with a = 1 to get (−1)(−1) = 1. 2. (a) −(a + b) = (−1)(a + b) = (−1)a + (−1)b = (−a) + (−b). (b) (−a) · (−b) = ((−1)a) · ((−1)b) = (−1)(−1)(ab) = ab. (c) Note that (−a)(−(1/a)) = a(1/a) = 1. (d) −(a/b) = (−1)(a(1/b)) = ((−1)a)(1/b) = (−a)/b. 3. (a) Add −5 to both sides of 2x + 5 = 8 and use (A2),(A4),(A3) to get 2x = 3. Then multiply both sides by 1/2 to get x = 3/2. (b) Write x2 − 2x = x(x − 2) = 0 and apply Theorem 2.1.3(b). Alternatively, note that x = 0 satisfies the equation, and if x = 0, then multiplication by 1/x gives x = 2. 7 9. 8 Bartle and Sherbert (c) Add −3 to both sides and factor to get x2 − 4 = (x − 2)(x + 2) = 0. Now apply 2.1.3(b) to get x = 2 or x = −2. (d) Apply 2.1.3(b) to show that (x − 1)(x + 2) = 0 if and only if x = 1 or x = −2. 4. Clearly a = 0 satisfies a · a = a. If a = 0 and a · a = a, then (a · a)(1/a) = a(1/a), so that a = a(a(1/a)) = a(1/a) = 1. 5. If (1/a)(1/b) is multiplied by ab, the result is 1. Therefore, Theorem 2.1.3(a) implies that 1/(ab) = (1/a)(1/b). 6. Note that if q ∈ Z and if 3q2 is even, then q2 is even, so that q is even. Hence, if (p/q)2 = 6, then it follows that p is even, say p = 2m, whence 2m2 = 3q2, so that q is also even. 7. If p ∈ N, there are three possibilities: for some m ∈ N ∪ {0}, (i) p = 3m, (ii) p = 3m + 1, or (iii) p = 3m + 2. In either case (ii) or (iii), we have p2 = 3h + 1 for some h ∈ N ∪ {0}. 8. (a) Let x = m/n, y = p/q, where m, n = 0, p, q = 0 are integers. Then x + y = (mq + np)/nq and xy = mp/nq are rational. (b) If s := x + y ∈ Q, then y = s − x ∈ Q, a contradiction. If t := xy ∈ Q and x = 0, then y = t/x ∈ Q, a contradiction. 9. (a) If x1 = s1 + t1 √ 2 and x2 = s2 + t2 √ 2 are in K, then x1 + x2 = (s1 + s2) + (t1 + t2) √ 2 and x1x2 = (s1s2 + 2t1t2) + (s1t2 + s2t1) √ 2 are also in K. (b) If x = s + t √ 2 = 0 is in K, then s − t √ 2 = 0 (why?) and 1 x = s − t √ 2 (s + t √ 2)(s − t √ 2) = s s2 − 2t2 − t s2 − 2t2 √ 2 is in K. (Use Theorem 2.1.4.) 10 (a) If c = d, then 2.1.7(b) implies a + c < b + d. If c < d, then a + c < b + c < b + d. (b) If c = d = 0, then ac = bd = 0. If c > 0, then 0 < ac by the Trichotomy Property and ac < bc follows from 2.1.7(c). If also c ≤ d, then ac ≤ ad < bd. Thus 0 ≤ ac ≤ bd holds in all cases. 11. (a) If a > 0, then a = 0 by the Trichotomy Property, so that 1/a exists. If 1/a = 0, then 1 = a·(1/a) = a·0 = 0, which contradicts (M3). If 1/a < 0, then 2.1.7(c) implies that 1 = a(1/a) < 0, which contradicts 2.1.8(b). Thus 1/a > 0, and 2.1.3(a) implies that 1/(1/a) = a. (b) If a < b, then 2a = a + a < a + b, and also a + b < b + b = 2b. Therefore, 2a < a + b < 2b, which, since 1 2 > 0 (by 2.1.8(c) and part (a)), implies that a < 1 2 (a + b) < b. 12. Let a = 1 and b = 2. If c = −3 and d = −1, then ac < bd. On the other hand, if c = −3 and d = −2, then bd < ac. (Many other examples are possible.) 10. Chapter 2 — The Real Numbers 9 13. If a = 0, then 2.1.8(a) implies that a2 > 0; since b2 ≥ 0, it follows that a2 + b2 > 0. 14. If 0 ≤ a < b, then 2.1.7(c) implies ab < b2. If a = 0, then 0 = a2 = ab < b2. If a > 0, then a2 < ab by 2.1.7(c). Thus a2 ≤ ab < b2. If a = 0, b = 1, then 0 = a2 = ab < b = 1. 15. (a) If 0 < a < b, then 2.1.7(c) implies that 0 < a2 < ab < b2. Then by Example 2.1.13(a), we infer that a = √ a2 < √ ab < √ b2 = b. (b) If 0 < a < b, then ab > 0 so that 1/ab > 0, and thus 1/a − 1/b = (1/ab)(b − a) > 0. 16. (a) To solve (x − 4)(x + 1) > 0, look at two cases. Case 1: x − 4 > 0 and x + 1 > 0, which gives x > 4. Case 2: x − 4 < 0 and x + 1 < 0, which gives x < −1. Thus we have {x : x > 4 or x < −1}. (b) 1 < x2 < 4 has the solution set {x : 1 < x < 2 or − 2 < x < −1}. (c) The inequality is 1/x − x = (1 − x)(1 + x)/x < 0. If x > 0, this is equiva- lent to (1 − x)(1 + x) < 0, which is satisfied if x > 1. If x < 0, then we solve (1 − x)(1 + x) > 0, and get −1 < x < 0. Thus we get {x : −1 < x < 0 or x > 1} (d) the solution set is {x : x < 0 or x > 1}. 17. If a > 0, we can take ε0 := a > 0 and obtain 0 < ε0 ≤ a, a contradiction. 18. If b < a and if ε0 := (a − b)/2, then ε0 > 0 and a = b + 2ε0 > b + ε0. 19. The inequality is equivalent to 0 ≤ a2 − 2ab + b2 = (a − b)2. 20. (a) If 0 < c < 1, then 2.1.7(c) implies that 0 < c2 < c, whence 0 < c2 < c < 1. (b) Since c > 0, then 2.1.7(c) implies that c < c2, whence 1 < c < c2. 21. (a) Let S := {n ∈ N : 0 < n < 1}. If S is not empty, the Well-Ordering Property of N implies there is a least element m in S. However, 0 < m < 1 implies that 0 < m2 < m, and since m2 is also in S, this is a contradiction to the fact that m is the least element of S. (b) If n = 2p = 2q − 1 for some p, q in N, then 2(q − p) = 1, so that 0 < q − p < 1. This contradicts (a). 22. (a) Let x := c − 1 > 0 and apply Bernoulli's Inequality 2.1.13(c) to get cn = (1 + x)n ≥ 1 + nx ≥ 1 + x = c for all n ∈ N, and cn > 1 + x = c for n > 1. (b) Let b := 1/c and use part (a). 23. If 0 < a < b and ak < bk, then 2.1.7(c) implies that ak + 1 < abk < bk + 1 so Induction applies. If am < bm for some m ∈ N, the hypothesis that 0 < b ≤ a leads to a contradiction. 24. (a) If m > n, then k := m − n ∈ N, so Exercise 22(a) implies that ck ≥ c > 1. But since ck = cm − n, this implies that cm > cn. Conversely, the hypothesis that cm > cn and m ≤ n lead to a contradiction. (b) Let b := 1/c and use part (a). 11. 10 Bartle and Sherbert 25. Let b := c1/mn. We claim that b > 1; for if b ≤ 1, then Exercise 22(b) implies that 1 < c = bmn ≤ b ≤ 1, a contradiction. Therefore Exercise 24(a) implies that c1/n = bm > bn = c1/m if and only if m > n. 26. Fix m ∈ N and use Mathematical Induction to prove that am + n = aman and (am)n = amn for all n ∈ N. Then, for a given n ∈ N, prove that the equalities are valid for all m ∈ N. Section 2.2 The notion of absolute value of a real number is defined in terms of the basic order properties of R. We have put it in a separate section to give it emphasis. Many students need extra work to become comfortable with manipulations involving absolute values, especially when inequalities are involved. We have also used this section to give students an early introduction to the notion of the ε-neighborhood of a point. As a preview of the role of ε-neighborhoods, we have recast Theorem 2.1.9 in terms of ε-neighborhoods in Theorem 2.2.8. Sample Assignment: Exercises 1, 4, 5, 6(a,b), 8(a,b), 9, 12(a,b), 15. Partial Solutions: 1. (a) If a ≥ 0, then |a| = a = √ a2; if a < 0, then |a| = −a = √ a2. (b) It suffices to show that |1/b| = 1/|b| for b = 0 (why?). If b > 0, then 1/b > 0 (why?), so that |1/b| = 1/b = 1/|b|. If b < 0, then 1/b < 0, so that |1/b| = −(1/b) = 1/(−b) = 1/|b|. 2. First show that ab ≥ 0 if an only if |ab| = ab. Then show that (|a| + |b|)2 = (a + b)2 if and only if |ab| = ab. 3. If x ≤ y ≤ z, then |x − y| + |y − z| = (y − x) + (z − y) = z − x = |z − x|. To establish the converse, show that y < x and y > z are impossible. For example, if y < x ≤ z, it follows from what we have shown and the given relationship that |x − y| = 0, so that y = x, a contradiction. 4. |x − a| < ε ⇐⇒ −ε < x − a < ε ⇐⇒ a − ε < x < a + ε. 5. If a < x < b and −b < −y < −a, it follows that a − b < x − y < b − a. Since a − b = −(b − a), the argument in 2.2.2(c) gives the conclusion |x − y| < b − a. The distance between x and y is less than or equal to b − a. 6. (a) |4x − 5| ≤ 13 ⇐⇒ −13 ≤ 4x − 5 ≤ 13 ⇐⇒ −8 ≤ 4x ≤ 18 ⇐⇒ −2 ≤ x ≤ 9/2. (b) |x2 − 1| ≤ 3 ⇐⇒ −3 ≤ x2 − 1 ≤ 3 ⇐⇒ −2 ≤ x2 ≤ 4 ⇐⇒ 0 ≤ x2 ≤ 4 ⇐⇒ −2 ≤ x ≤ 2. 7. Case 1: x ≥ 2 ⇒ (x + 1) + (x − 2) = 2x − 1 = 7, so x = 4. Case 2: −1 < x < 2 ⇒ (x + 1) + (2 − x) = 3 = 7, so no solution. Case 3: x ≤ −1 ⇒ (−x − 1) + (2 − x) = −2x + 1 = 7, so x = −3. Combining these cases, we get x = 4 or x = −3. 12. Chapter 2 — The Real Numbers 11 8. (a) If x > 1/2, then x + 1 = 2x − 1, so that x = 2. If x ≤ 1/2, then x + 1 = −2x + 1, so that x = 0. There are two solutions {0, 2}. (b) If x ≥ 5, the equation implies x = −4, so no solutions. If x < 5, then x = 2. 9. (a) If x ≥ 2, the inequality becomes −2 ≤ 1. If x ≤ 2, the inequality is x ≥ 1/2, so this case contributes 1/2 ≤ x ≤ 2. Combining the cases gives us all x ≥ 1/2. (b) x ≥ 0 yields x ≤ 1/2, so that we get 0 ≤ x ≤ 1/2. x ≤ 0 yields x ≥ −1, so that −1 ≤ x ≤ 0. Combining cases, we get −1 ≤ x ≤ 1/2. 10. (a) Either consider the three cases: x < −1, −1 ≤ x ≤ 1 and 1 < x; or, square both sides to get −2x > 2x. Either approach gives x < 0. (b) Consider the three cases x ≥ 0, − 1 ≤ x < 0 and x < − 1 to get − 3/2 < x < 1/2. 11. y = f(x) where f(x) := −1 for x < 0, f(x) := 2x − 1 for 0 ≤ x ≤ 1, and f(x) := 1 for x > 1. 12. Case 1: x ≥ 1 ⇒ 4 < (x + 2) + (x − 1) < 5, so 3/2 < x < 2. Case 2: −2 < x < 1 ⇒ 4 < (x + 2) + (1 − x) < 5, so there is no solution. Case 3: x < −2 ⇒ 4 < (−x − 2) + (1 − x) < 5, so −3 < x < −5/2. Thus the solution set is {x : −3 < x < −5/2 or 3/2 < x < 2}. 13. |2x − 3| < 5 ⇐⇒ −1 < x < 4, and |x + 1| > 2 ⇐⇒ x < −3 or x > 1. The two inequalities are satisfied simultaneously by points in the intersection {x : 1 < x < 4}. 14. (a) |x| = |y| ⇐⇒ x2 = y2 ⇐⇒ (x − y)(x + y) = 0 ⇐⇒ y = x or y = −x. Thus {(x, y) : y = x or y = −x}. (b) Consider four cases. If x ≥ 0, y ≥ 0, we get the line segment joining the points (0, 1) and (1, 0). If x ≤ 0, y ≥ 0, we get the line segment joining (−1, 0) and (0, 1), and so on. (c) The hyperbolas y = 2/x and y = −2/x. (d) Consider four cases corresponding to the four quadrants. The graph consists of a portion of a line segment in each quadrant. For example, if x ≥ 0, y ≥ 0, we obtain the portion of the line y = x − 2 in this quadrant. 15. (a) If y ≥ 0, then −y ≤ x ≤ y and we get the region in the upper half-plane on or between the lines y = x and y = −x. If y ≤ 0, then we get the region in the lower half-plane on or between the lines y = x and y = −x. (b) This is the region on and inside the diamond with vertices (1, 0), (0, 1), (−1, 0) and (0, −1). 16. For the intersection, let γ be the smaller of ε and δ. For the union, let γ be the larger of ε and δ. 17. Choose any ε > 0 such that ε < |a − b|. 18. (a) If a ≤ b, then max{a, b} = b = 1 2 [a + b + (b − a)] and min{a, b} = a = 1 2 [a + b − (b − a)]. (b) If a = min {a, b, c}, then min{min{a, b}, c} = a = min{a, b, c}. Similarly, if b or c is min{a, b, c}. 13. 12 Bartle and Sherbert 19. If a ≤ b ≤ c, then mid{a, b, c} = b = min{b, c, c} = min{max{a, b}, max{b, c}, max{c, a}}. The other cases are similar. Section 2.3 This section completes the description of the real number system by introducing the fundamental completeness property in the form of the Supremum Property. This property is vital to real analysis and students should attain a working under- standing of it. Effort expended in this section and the one following will be richly rewarded later. Sample Assignment: Exercises 1, 2, 5, 6, 9, 10, 12, 14. Partial Solutions: 1. Any negative number or 0 is a lower bound. For any x ≥ 0, the larger number x + 1 is in S1, so that x is not an upper bound of S1. Since 0 ≤ x for all x ∈ S1, then u = 0 is a lower bound of S1. If v > 0, then v is not a lower bound of S1 because v/2 ∈ S1 and v/2 < v. Therefore inf S1 = 0. 2. S2 has lower bounds, so that inf S2 = 0, but that inf S2 does not belong to S2. S2 does not have upper bounds, so that sup S2 does not exists. 3. Since 1/n ≤ 1 for all n ∈ N, then 1 is an upper bound for S3. But 1 is a member of S3, so that 1 = sup S3. (See Exercise 7 below.) 4. sup S4 = 2 and inf S4 = 1/2. (Note that both are members of S4.) 5. It is interesting to compare algebraic and geometric approaches to these problems. (a) inf A = −5/2, sup A does not exist, (b) sup B = 2, inf B = −1, (c) sup C = 1, inf B does not exist, (d) sup D = 1 + √ 6, inf D = 1 − √ 6. 6. If S is nonempty and bounded below, then S := {−s : s ∈ S} is bounded above, so that u := sup S exists. If v = s for all s ∈ S, then −v ≥ −s for all s ∈ S, so that −v ≥ u, and hence v ≤ −u. Thus inf S = −u. 7. Let u ∈ S be an upper bound of S. If v is another upper bound of S, then u ≤ v. Hence u = sup S. 8. If t > u and t ∈ S, then u is not an upper bound of S. 9. Let u := sup S. Since u is an upper bound of S, so is u + 1/n for all n ∈ N. Since u is the supremum of S and u − 1/n < u, then there exists s0 ∈ S with u − 1/n < s0, whence u − 1/n is not an upper bound of S. 10. Let u := sup A, v := sup B and w := sup{u, v}. Then w is an upper bound of A ∪ B, because if x ∈ A, then x ≤ u ≤ w, and if x ∈ B, then x ≤ v ≤ w. If z is any upper bound of A ∪ B, then z is an upper bound of A and of B, so that u ≤ z and v ≤ z. Hence w ≤ z. Therefore, w = sup(A ∪ B). 11. Since sup S is an upper bound of S, it is an upper bound of S0, and hence sup S0 ≤ sup S. 12. Consider two cases. If u ≥ s∗, then u = sup(S ∪ {u}). If u < s∗, then there exists s ∈ S such that u < s ≤ s∗, and that s∗ = sup(S ∪ {u}). 13. If S1 := {x1}, show that x1 = sup S1. If Sk := {x1, . . . , xk} is such that sup Sk ∈ Sk, then preceding exercise implies that sup{x1, . . . , xk, xk + 1} is the larger of sup Sk and xk+1 and so is in Sk + 1. 14. If w = inf S and ε > 0, then w + ε is not a lower bound so that there exists t in S such that t < w + ε. If w is a lower bound of S that satisfies the stated condition, and if z > w, let ε := z − w > 0. Then there is t in S such that t < w + ε = z, so that z is not a lower bound of S. Thus, w = inf S. Section 2.4 This section exhibits how the supremum is used in practice, and contains some important properties of R that will often be used later. The Archimedean Proper- ties 2.4.3–2.4.6 and the Density Properties 2.4.8 and 2.4.9 are the most significant. The exercises also contain some results that will be used later. Sample Assignment: Exercises 1, 2, 4(b), 5, 7, 10, 12, 13, 14. Partial Solutions: 1. Since 1 − 1/n < 1 for all n ∈ N, the number 1 is an upper bound. To show that 1 is the supremum, it must be shown that for each ε > 0 there exists n ∈ N such that 1 − 1/n > 1 − ε, which is equivalent to 1/n < ε. Apply the Archimedean Property 2.4.3 or 2.4.5. 2. inf S = −1 and sup S = 1. To see the latter note that 1/n − 1/m < 1 for all m, n ∈ N. On the other hand if ε > 0 there exists m ∈ N such that 1/m < ε, so that 1/1 − 1/m > 1 − ε. 3. Suppose that u ∈ R is not the supremum of S. Then either (i) u is not an upper bound of S (so that there exists s1 ∈ S with u < s1, whence we take n ∈ N with 1/n < s1 − u to show that u + 1/n is not an upper bound of S), or (ii) there exists an upper bound u1 of S with u1 < u (in which case we take 1/n < u − u1 to show that u − 1/n is not an upper bound of S). 4. (a) Let u := sup S and a > 0. Then x ≤ u for all x ∈ S, whence ax ≤ au for all x ∈ S, whence it follows that au is an upper bound of aS. If v is another upper bound of aS, then ax ≤ v for all x ∈ S, whence x ≤ v/a for all x ∈ S, showing that v/a is an upper bound for S so that u ≤ v/a, from which we conclude that au ≤ v. Therefore au = sup(aS). The statement about the infimum is proved similarly. 15. 14 Bartle and Sherbert (b) Let u := sup S and b < 0. If x ∈ S, then (since b < 0) bu ≤ bx so that bu is a lower bound of bS. If v ≤ bx for all x ∈ S, then x ≤ v/b (since b < 0), so that v/b is an upper bound for S. Hence u ≤ v/b whence v ≤ bu. Therefore bu = inf(bS). 5. If x ∈ S, then 0 ≤ x ≤ u, so that x2 ≤ u2 which implies sup T ≤ u2. If t is any upper bound of T, then x ∈ S implies x2 ≤ t so that x ≤ √ t. It follows that u ≤ √ t, so that u2 ≤ t. Thus u2 ≤ sup T. 6. Let u := sup f(X). Then f(x) ≤ u for all x ∈ X, whence a + f(x) ≤ a + u for all x ∈ X, whence sup{a + f(x) : x ∈ X} ≤ a + u. If w < a + u, then w − a < u, so that there exists xw ∈ X with w − a < f(xw), whence w < a + f(xw), and thus w is not an upper bound for {a + f(x) : x ∈ X}. 7. Let u := sup S, v := sup B, w := sup(A + B). If x ∈ A and y ∈ B, then x + y ≤ u + v. Now, fix y ∈ B and note that w − y for all x ∈ A; thus w − y is an upper bound for A so that u ≤ w − y. Then y ≤ w − u for all y ∈ B, so v ≤ w − u and hence u + v ≤ w. Combining these inequalities, we have w = u + v. 8. If u := sup f(X) and v := sup g(X), then f(x) ≤ u and g(x) ≤ v for all x ∈ X, whence f(x) + g(x) ≤ u + v for all x ∈ X. Thus u + v is an upper bound for the set {f(x) + g(x) : x ∈ X}. Therefore sup{f(x) + g(x) : x ∈ X} ≤ u + v. 9. (a) f(x) = 2x + y for all x ∈ X and y ∈ Y, so that f(x) ≤ sup{F(x) : x ∈ X}. (b) g(y) = y, sup{g(y) : y ∈ Y } = 1. 10. (a) f(x) = 1 for x ∈ X. (b) g(y) = 0 for y ∈ Y . 11. If x ∈ X, y ∈ Y , then g(y) ≤ h(x, y) ≤ f(x). If we fix y ∈ Y and take the infimum over x ∈ X, then we get g(y) ≤ inf{f(x) : x ∈ X} for each y ∈ Y . Now take the supremum over y ∈ Y . 12. Let S := {h(x, y) : x ∈ X, y ∈ Y }. We have h(x, y) ≤ F(x) for all x ∈ X, y ∈ Y so that sup S ≤ sup{F(x) : x ∈ X}. If w < sup{F(x) : x ∈ X}, then there exists x0 ∈ X with w < F(x0) = sup {h(x0, y) : y ∈ Y }, whence there exists y0 ∈ Y with w < h(x0, y0). Thus w is not an upper bound of S, and so w < sup S. Since this is true for any w such that w < sup{F(x) : x ∈ X}, we conclude that sup{F(x) : x ∈ X} ≤ sup S. 13. If x ∈ Z, take n := x + 1. If x /∈ Z, we have two cases: (i) x > 0 (which is covered by Cor. 2.4.6), and (ii) x < 0. In case (ii), let z := −x and use 2.4.6. If n1 < n2 are integers, then n1 ≤ n2 − 1 so the sets {y : n1 − 1 ≤ y < n1} and {y : n2 − 1 ≤ y < n2} are disjoint; thus the integer n such that n − 1 ≤ x < n is unique. 14. Note that n < 2n (whence 1/2n < 1/n) for any n ∈ N. 15. Let S3 := {s ∈ R : 0 ≤ s, s2 < 3}. Show that S3 is nonempty and bounded by 3 and let y := sup S3. If y2 < 3 and 1/n < (3 − y2)/(2y + 1) show that 16. Chapter 2 — The Real Numbers 15 y + 1/n ∈ S3. If y2 > 3 and 1/m < (y2 − 3)/2y show that y − 1/m ∈ S3. Therefore y2 = 3. 16. Case 1: If a > 1, let Sa := {s ∈ R : 0 ≤ s, s2 < a}. Then Sa is nonempty and bounded above by a and let z := sup Sa. Now show that z2 = a. Case 2: If 0 < a < 1, there exists k ∈ N such that k2a > 1 (why?). If z2 = k2a, then (z/k)2 = a. 17. Consider T := {t ∈ R : 0 ≤ t, t2 < 2}. If t > 2, then t3 > 2 so that t /∈ T. Hence y := sup T exists. If y3 < 2, choose 1/n < (2 − y3)/(3y2 + 3y + 1) and show that (y + 1/n)3 < 2, a contradiction, and so on. 18. If u < 0 < v, then we can take r = 0. If u < v ≤ 0, we apply 2.4.8 to obtain a rational number between −v and −u. 19. There exists r ∈ Q such that x/u < r < y/u. Section 2.5 Another important consequence of the Supremum Property of R is the Nested Intervals Property 2.5.2. It is an interesting fact that if we assume the validity of both the Archimedean Property 2.4.3 and the Nested Intervals Property, then we can prove the Supremum Property. Hence these two properties could be taken as the completeness axiom for R. However, establishing this logical equivalence would consume valuable time and not significantly advance the study of real anal- ysis, so we will not do so. (There are other properties that could be taken as the completeness axiom.) The discussion of binary and decimal representations is included to give the student a concrete illustration of the rather abstract ideas developed to this point. However, this material is not vital for what follows and can be omitted or treated lightly. We have kept this discussion informal to avoid getting buried in technical details that are not central to the course. Sample Assignment: Exercises 1, 3, 4, 5, 6, 7, 8, 10, 11. Partial Solutions: 1. Note that [a, b] ⊆ [a , b] if and only if a ≤ a ≤ b ≤ b . 2. S has an upper bound b and a lower bound a if and only if S is contained in the interval [a, b]. 3. Since inf S is a lower bound for S and sup S is an upper bound for S, then S ⊆ IS. Moreover, if S ⊆ [a, b], then a is a lower bound for S and b is an upper bound for S, so that [a, b] ⊇ IS. 4. Because z is neither a lower bound or an upper bound of S so there exists xz ∈ S such that xz ≤ z. Also z is not an upper bound of S so there exists yz ∈ S such that z ≤ yz. Since z belongs to [xz, yz], it follows from the property (1) that z ∈ S. Introduction to Real Analysis 4th Edition Bartle Solutions Manual Full Download: This sample only, Download all chapters at: alibabadownload.com

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